Recent Advancement in Modeling of Wave Propagation and Breaking Waves in Surf Zone

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Abstract
Modeling of shallow water wind-waves has been a key component of coastal wave research. Numerical results of these wave models play important roles in a wide range of applications, including the design of commercial and military operations, and the decision-making processes in coastal zone management and hazard mitigation. The purpose of this paper is to give a brief review of the recent progress in mathematical/numerical modeling of wind-waves propagating from deep water into a surf zone. The primary focus of the review is on the depth-integrated wave models. However, the modeling of breaking waves using the Reynolds Averaged Navier-Stokes equations is also discussed.

1. Introduction

Every coastal or ocean engineering study such as a beach renourishment project or a harbor design, requires the information of wave conditions in the area of interest. Usually, wave characteristics are collected offshore and it is necessary to transfer these offshore data on wave heights and wave propagation direction to the project site. The increasing demands for accurate design wave conditions have resulted in significant advancement of wave transformation models during the last two decades.

In the early 1960's the wave ray tracing method was a common tool for consulting engineers to estimate wave characteristics at a design site. Today, powerful computers have provided coastal engineers with the opportunity to employ more sophisticated numerical models for wave environment assessment. However, these numerical models are still based on simplified governing equations, boundary conditions and numerical schemes, imposing different restrictions to practical applications. The computational effort required for solving a wave propagation problem exactly by taking all physical processes, which involve many different temporal and spatial scales, into account is still not practical.

To date, two basic kinds of numerical wave models can be distinguished: phase-resolving models, based on vertically integrated, time-dependent mass and momentum balance equations and phase-averaged models, which are based on a spectral energy balance equation. The application of phase-resolving models is limited to relatively small areas O (1 km) while phase-averaged models do not require fine resolution and can be used in much larger areas.

The more recent research has been focused on the development of unified models, which can describe transient fully nonlinear wave propagation from deep water to shallow water over large areas. Furthermore, significant progress has also been made in the modeling of the wave-breaking process and in the simulation of the wave and structure interaction.

2. Wave Propagation Models

2.1 Ray approximation

Efforts for reducing the computational efforts are necessary and have been sought by reducing the dimension of the computational domain. Moreover, continuing efforts have been made to construct a unified model that can propagate wave from deep water into shallow water, even into the surf zone. The forerunner of this kind of effort is the ***ray approximation*** for ***infinitesimal waves*** propagating over bathymetry that vary slowly over horizontal distances much longer than the local wavelength. In this approximation, one first finds wave rays by adopting the geometrical optic theory, which defines the wave ray as a curve tangential to the wave number vector. One then calculates the spatial variation of the wave envelope along the rays by invoking the principle of conservation of energy. Numerical discretization can be done in steps along a ray not necessarily small in comparison with a typical wavelength. Since the ray approximation does not allow wave energy flux across a wave ray, it fails near the caustics or the focal regions, where neighboring wave rays intersect; diffract and possibly nonlinearity are important. While ad hoc numerical methods for local remedies are available, it is not always convenient to implement them in practice.

2.2 Mild-slope equation

Within the frame work of linear wave theory, an improvement to the ray approximation was first suggested by Eckart (1952) and was later rederived by Berkhoff (1972, 1976), who proposed a two-dimensional theory which can deal with large regions of refraction and diffraction. The underlying assumption of the theory is that evanescent modes are not important for waves propagating over a slowly varying bathymetry, except in the immediate vicinity of a three-dimensional obstacle. For a monochromatic wave with frequency ω and free surface displacement η, it is reasonable to express the velocity potential, which formally represents the propagating model only, as:

$$\phi = \frac{-ig\eta}{\omega}\frac{\cosh k(z+h)}{\cosh kh}e^{-i\omega t} \tag{1}$$

where $k(x,y)$ and $h(x,y)$ vary slowly in the horizontal directions, x and y, according to the linear frequency dispersion relation, $\omega^2 = gk\tanh kh$, where g is the gravitational acceleration. By a perturbation argument one can show that the free surface displacement η must satisfy the following equation:

$$\nabla\cdot(CC_g\nabla\eta) + \frac{\omega^2}{g}\eta = 0, \tag{2}$$

where C and C_g are the local phase and group velocities of a plane progressive wave. The elliptic-type partial differential equation, (2), is asymptotically valid for sufficiently small δ ($=|\nabla h|/kh$ to leading order) and is known as the ***mild-slope equation***. An indication of its versatility can be seen in two limits. For long waves in shallow water the limit of (2) at $kh \ll 1$ reduces to the well-known linear shallow-water equation that is valid even if $\delta = O(1)$. On the other hand, if the depth is a constant or for short waves in deep water ($kh \gg 1$), (2) reduces to the Helmholtz equation where k satisfies the dispersion relation. Both limits can be used to calculate diffraction legitimately. Thus, the mild slope-equation should be a good interpolation for all kh and is suitable for propagating wave from deep water to shallow water as long as the linearization is acceptable. A similar mild-slope equation for waves propagating over gradually varying currents has also been derived (e.g. Liu, 1990).

2.3 Parabolic approximation

In applying the mild-slope equation to a large region in coastal zone, one encounters the difficulty of specifying boundary conditions along the shoreline, which are essential for solving the elliptic-type mild-slope equation. The difficulty arises because the location of the breaker can not be determined *a priori*. A remedy to this problem is to apply the ***parabolic approximation*** to the mild-slope equation. For essentially forward propagation problems, the so-called parabolic approximation expands the validity of the ray theory by allowing wave energy "diffuse" across the wave "ray". Therefore, the effects of diffraction have been approximately included in the parabolic approximation. Although the parabolic approximation has been used primarily for forward propagation, adopting an iterative procedure can also include weakly backward propagation (e.g. Liu and Tsay 1983, Chen and Liu 1994).

2.4 Depth-integrated Wave Models

It is well-known that in the fairly shallow water, where both nonlinearity and frequency dispersion are weak and are in the same order of magnitude, the ***standard Boussinesq equations*** for variable depth are adequate wave propagation model (Peregrine 1967).

$$\eta_t + \nabla\cdot[(\eta+h)\bar{u}] = 0 \tag{3}$$

$$\bar{u}_t + \frac{1}{2}\nabla|\bar{u}|^2 + g\nabla\eta + \left\{\frac{h^2}{6}\nabla(\nabla\cdot\bar{u}_t) - \frac{h}{2}\nabla(\nabla\cdot(h\bar{u}_t))\right\} = 0 \tag{4}$$

in which $\bar{u}$ is the depth-averaged velocity, η the free surface displacement; h the still water depth, $\nabla = (\partial/\partial x, \partial/\partial y)$ the horizontal gradient operator, g the gravitational acceleration; and subscript t the partial derivative with respect to time. Boussinesq equations can be recast into similar equations in terms of either the velocity on the bottom or the velocity on the free surface. While the dispersion relationship and the wave speed associated with these equations differ slightly, the order of magnitude of accuracy of these equations remains the same. Numerical results based on the standard Boussinesq equations or the equivalent formulations have been shown to give predictions that compared quite well with field data (Elgar and Guza 1985) and laboratory data (Goring 1978, Liu *et al.* 1985).

Because it is required that both frequency dispersion and nonlinear effects are weak, the standard Boussinesq equations are not applicable to very shallow water depth, where the nonlinearity becomes more important than the frequency dispersion, and to the deep water depth, where the frequency dispersion is of order one. The standard Boussinesq equations written in terms of the depth-averaged velocity break down when the depth is greater than one-fifth of the equivalent deep-water wavelength. For many engineering applications, where the incident wave energy spectrum consists of many frequency components, a lesser depth restriction is desirable. Furthermore, when the Boussinesq equations are solved numerically, high frequency oscillations with wavelengths related to the grid size could cause instability. To extend the applications to shorter waves (or deeper water depth) many modified forms of Boussinesq-type equations have been introduced (e.g. Madsen *et al.* 1991, Nwogu 1993, Chen and Liu 1995). Although the methods of derivation are different, the resulting dispersion relations of the linear components of these modified Boussinesq equations are similar, and may be viewed as a slight modification of the (2,2) Pade approximation of the full dispersion relation for linear water wave (Witting 1984). The depth-integrated continuity equation and momentum equations can be expressed in terms of the free surface displacement η and $\boldsymbol{u}_\alpha$, the horizontal velocity vector at the water depth $z=z_\alpha$, can be expressed as:

$$\eta_t + \nabla\cdot[(\eta+h)\boldsymbol{u}_\alpha] + \nabla\cdot\left\{\left(\frac{z_\alpha^2}{2}-\frac{h^2}{6}\right)h\nabla(\nabla\cdot\boldsymbol{u}_\alpha) + \left(z_\alpha+\frac{h}{2}\right)h\nabla(\nabla\cdot h\boldsymbol{u}_\alpha)\right\} = 0 \qquad (5)$$

$$\boldsymbol{u}_{\alpha t} + \frac{1}{2}\nabla|\boldsymbol{u}_\alpha|^2 + g\nabla\eta + z_\alpha\left\{\frac{1}{2}z_\alpha\nabla(\nabla\cdot\boldsymbol{u}_{\alpha t}) + \nabla(\nabla\cdot(h\boldsymbol{u}_{\alpha t}))\right\} = 0 \qquad (6)$$

It has been demonstrated that with optimal choice of, $z_\alpha = -0.531h$ the ***modified Boussinesq equations*** are able to simulate wave propagation from deep water to shallow water including the wave-current interaction (Chen *et al.* 1998). It should be also pointed out that the convective acceleration in the momentum equation (4) and (6) has been written in the conservative form. One could replace them by the non-conservative form, i.e. $\overline{\boldsymbol{u}}\cdot\nabla\overline{\boldsymbol{u}}$ and $\boldsymbol{u}_\alpha\cdot\nabla\boldsymbol{u}_\alpha$, respectively, without changing the order of magnitude of accuracy of the model equations.

Despite of the success of the modified Boussinesq equations in intermediate and deep water, these equations are still restricted to weakly nonlinearity. As waves approach shore, wave height increases due to shoaling and wave breaks on most of gentle natural beaches. The wave-height to water depth ratios associated with this physical process become too high for the Boussinesq approximation. The appropriate model equation for the leading order solution should be the nonlinear shallow water equation. Of course this restriction can be readily removed by eliminating the weak nonlinearity assumption (e.g. Liu 1994, Wei *et al.* 1995). Strictly speaking, these fully nonlinear equations can no longer be called Boussinesq-type equations since the nonlinearity is not in balance with the frequency dispersion, which violates the spirit of the original Boussinesq assumption.

The fully nonlinear but weakly dispersive wave equations have been presented by many researchers and can be written as (Liu 1994):

$$\eta_t + \nabla\cdot\left\{(h+\eta)\left[u_\alpha + \left(z_\alpha + \frac{1}{2}(h-\eta)\right)\nabla(\nabla\cdot(hu_\alpha))\right.\right.$$
$$\left.\left. + \left(\frac{1}{2}z_\alpha^2 - \frac{1}{6}\left(h^2 - h\eta + \eta^2\right)\right)\nabla(\nabla\cdot u_\alpha)\right]\right\} = 0 \qquad (7)$$

$$\boldsymbol{u}_\alpha + \frac{1}{2}\nabla|\boldsymbol{u}_\alpha|^2 + g\nabla\eta + z_\alpha\left\{\frac{1}{2}z_\alpha\nabla(\nabla\cdot\boldsymbol{u}_\alpha)\right.$$
$$\left. + \nabla(\nabla\cdot(h\boldsymbol{u}_\alpha))\right\} + \nabla\left\{\frac{1}{2}\left(z_\alpha^2 - \eta^2\right)(\boldsymbol{u}_\alpha\cdot\nabla)(\nabla\cdot\boldsymbol{u}_\alpha)\right.$$
$$\left. + \frac{1}{2}\left[\nabla\cdot(h\boldsymbol{u}_\alpha) + \eta\nabla\cdot\boldsymbol{u}_\alpha\right]^2\right\} + \nabla\left\{(z_\alpha - \eta)(\boldsymbol{u}_\alpha\cdot\nabla)(\nabla\cdot(h\boldsymbol{u}_\alpha))\right.$$
$$\left. - \eta\left[\frac{1}{2}\eta\nabla\cdot\boldsymbol{u}_\alpha + \nabla\cdot(h\boldsymbol{u}_\alpha)\right]\right\} = 0 \qquad (8)$$

These equations are the statements of conservation of mass and momentum respectively. They are derived without making any approximation on the nonlinearity. Therefore, if one were to replace the conservative form of the inertia term, $\nabla|\boldsymbol{u}_\alpha|^2/2$, by $\boldsymbol{u}_\alpha\cdot\nabla\boldsymbol{u}_\alpha$ in the momentum equation, (8), additional higher order terms must be added to maintain the order of magnitude in accuracy. It is straightforward to show the conventional Boussinesq equations, (3) and (4), and the modified Boussinesq equations, (5) and (6), are the subsets of the unified model equations shown in (7) and (8).

2.5 Energy dissipation

In the previous sections all the wave theories have been developed based on the assumption that no energy dissipation occurs during the wave transformation process. However, in most coastal problems the effects of energy dissipation, such as bottom friction and wave breaking may become significant. In the numerical models based on Boussinesq-type equations, adding a new term to the depth-integrated momentum equation parameterizes the wave breaking process. While Zelt (1991), Karambas and Koutitas (1992) and Kennedy *et al.* (2000) used the eddy viscosity model, Brocchini *et al.* (1992) and Schäffer *et al.* (1993) employed a more complicated roller model based on the surface roller concept for spilling breakers. In the roller model the instantaneous roller thickness at each point and the orientation of the roller must be prescribed. Furthermore, in both approximations incipient breaking has to be determined making certain assumptions. By adjusting parameters associated with the breaking models, results of these models all showed very reasonable agreement with the respective laboratory data for free surface profiles. However, these models are unlikely to produce accurate solutions for the velocity field or to determine spatial distributions of the turbulent kinetic energy and therefore, more specific models on breaking waves are needed.

2.6 Reynolds Averaged Navier-Stokes (RANS) Equations model for Breaking Waves

Numerical modeling of three-dimensional breaking waves is extremely difficult. Several challenging tasks must be overcome. First of all, one must be able to track accurately the free surface location during the wave breaking process so that the near surface dynamics is captured. Secondly, one must properly model the physics of turbulence production, transport and dissipation throughout the entire wave breaking process. Thirdly, one needs to overcome the huge demand in computational resources.

There have been some successful two-dimensional results. For instance, the marker and cell (MAC) method (e.g., Johnson, *et al.* 1994) and the volume of fluid method (VOF) (e.g., Ng and Kot 1992, Lin and Liu 1998a) have been used to calculate two-dimensional breaking waves. The Reynolds Averaged Navier-Stokes (RANS) equations coupled with a second-order k - ε turbulence closure model have been shown to describe two-dimensional spilling and plunging breaking waves in surf zones (Lin and Liu 1998a,b). On the other hand, the Large Eddy Simulation (LES) approach has also been applied successfully for open channel flows where the free surface does not break (e.g., Hodges 1997). However, very little research has been reported for simulating three-dimensional breaking waves. Kawamura (1998) presented numerical models for three-dimensional ship waves by simulating a uniform free-surface flow passing a vertical cylinder. The dynamic process of a quasi-steady state ship waves is quite different from that of the breaking waves in surf zone.

The RANS model described above has been verified by comparing numerical results with either experimental data or analytical solutions.

For non-breaking waves, numerical models can accurately generate and propagate solitary waves as well as periodic waves. The numerical model can also simulate the overturning of a surface jet as the initial phase of the plunging wave breaking processes. For both spilling and plunging breaking waves on a beach the numerical results have been verified by laboratory data carefully performed by Ting and Kirby (1994, 1995). The detailed descriptions of the numerical results and their comparison with experimental data can be found in Liu and Lin (1997) and Lin and Liu (1998a,b). The overall agreement between numerical solutions and experimental data was very good. Although they are not shown here, the numerical results also have been used to explain the generation and transport of turbulence and vorticity throughout the wave breaking process. The vertical profiles of the eddy viscosity are obtained through out the surf zone. The surf similarity has been observed. Moreover, the model has also been used to demonstrate the different diffusion processes of pollutant release inside and outside of the surf zone (Lin and Liu 1998b)

At the present time, the model is limited to two-dimensional idealization. It is highly desirable to extend its capability to handle three-dimensionality.

3. Concluding Remarks

This paper has provided a brief review of the recent advancement in modeling of wave propagation from deep water to surf zone. Our understanding of the roles of the nonlinearity and frequency dispersion in wave propagation has improved significantly and a unified depth-integrated wave model has been obtained. However, our knowledge on the wave breaking process is still in its infancy. More research effort in this area is certainly needed. Moreover, although the wind effects on the surf zone dynamics is not discussed in this article, it is by no means unimportant. In fact, strong wind conditions could result in higher setup as well as stronger longshore currents.

At present, most of operational models are the coupling of spectral wave models for a large region and linear phase-resolving wave models for smaller sub-regions (e.g., Allard. *et al.* 1998). The nonlinear models discussed in this article have not been integrated into the operational system, because of the relatively large requirement in the computational efforts. Therefore, more research is also needed in search for robust, accurate and efficient numerical methods for solving the nonlinear wave models with wave breaking and wind effects.

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